Navigation Tracking with Multiple Baselines Part I: High-Level Theory and System Concepts[1]

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***Abstract*— Delta Differential One-Way Ranging (ΔDOR) and Same Beam Interferometry (SBI) are deep space tracking techniques that use two widely separated ground antennas, known as a baseline, to simultaneously track a transmitting spacecraft to measure the time difference between signals arriving at the two stations. Errors are introduced into the delay measurements when the radio waves pass through the solar plasma and the Earth's atmosphere, and also due to clock bias and clock instability of the ground stations. These errors can be eliminated or calibrated by tracking a quasar in the angular vicinity of the spacecraft (ΔDOR), or by tracking another close-by spacecraft whose trajectory/orbit is accurately known (SBI). Both ΔDOR and SBI uses this double-differencing of signal arrival time to eliminate the aforementioned error sources, and to generates highly accurate angular measurements with respect to the baseline.**

The three Deep Space Network (DSN) sites cover three approximately equally-spaced longitudes to provide near-continuous coverage of deep space. Spacecraft occasionally see two DSN sites simultaneously, but never three. Current DSN's ΔDOR and SBI techniques are based on one baseline of two sites. But recent additions of non-DSN deep space antennas and increased cross-support collaborations between space agencies allow spacecraft seeing more than one baselines simultaneously.

In this paper, we consider simultaneous SBI of two or more baselines that share one common ground station. We show that under certain condition, precise pointing vector between the common ground station and the spacecraft can be computed using simultaneous ΔDOR measurements from the two baselines. When there is another spacecraft in the vicinity of the first spacecraft, precise pointing vector of the second spacecraft can be derived from the simultaneous SBI measurements. Also, precise angular distance between the two spacecraft can be computed in real-time. We expect these new data types could enhance ground antenna pointing, and deep space spacecraft trajectory estimation and orbit determination.

This technique can have near-Earth applications. We describe a system concept that detects and locates dead and non-cooperatively spacecraft in the Geostationary Orbit (GEO). This can be done by making use of an existing GEO satellite with accurately known location as a reference, and/or by placing a dedicated "reference" spacecraft into an eccentric geosynchronous orbit over a region of interest (e.g. above North America). By adjusting the orbit, the "reference" spacecraft can sweep through the sky back-and-forth in the vicinity of the GEO over the region. In this way, the reference spacecraft can be close to any "static" GEO targets along its path. Using the multi-static radar approach, the ground transmitting radar illuminates both the reference and target spacecraft, and the ground receiving radars measure the different time-delays of signal arrival. Applying a variant of the aforementioned simultaneous SBI scheme, the time-delay measurements can be used to compute the precise relative position of the target spacecraft with respect to the reference spacecraft, whose position can be accurately estimated using the weak Global Positioning Satellite (GPS) signals.

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1. INTRODUCTION

Delta Differential One-Way Ranging (ΔDOR) and Same Beam Interferometry (SBI) are operational deep space tracking techniques that use two widely separated ground antennas, known as a baseline, to simultaneously track a transmitting spacecraft to measure the time difference between signals arriving at the two stations. Errors are

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introduced into the delay measurements when the radio waves pass through the solar plasma and the Earth's atmosphere, and also due to clock bias and clock instability of the ground stations. These errors can be eliminated or calibrated by tracking a quasar in the angular vicinity of the spacecraft (ΔDOR), or by tracking another close-by spacecraft whose trajectory/orbit is accurately known (SBI). Both ΔDOR and SBI uses this double-differencing of signal arrival time to eliminate the aforementioned error sources, and to generates highly accurate angular measurements with respect to the baseline. ΔDOR and SBI were extensively used in the Chang'E-3 Mission for the Lander's descent phase and for the rover's precise positioning [1][2]. SBI was also used in the Phoenix spacecraft's Mars approach navigation [3]. Detailed discussion on ΔDOR and SBI system design and operation can be found in [4].

The three Deep Space Network (DSN) sites cover three approximately equally-spaced longitudes to provide near-continuous coverage of deep space. Spacecraft occasionally see two DSN sites simultaneously, but never three. Current DSN's ΔDOR and SBI techniques are based on one baseline of two sites. Right Ascension and Declination (RA/DEC) is estimated using measurements from multiple station passes, and the process is non-real-time. However recent additions of non-DSN deep space antennas and increased cross-support collaborations between space agencies allow spacecraft seeing more than one baselines simultaneously. For example, with the addition of a non-DSN Malargue antenna, a DSN Goldstone antenna can form two simultaneous long baselines: Goldstone-Madrid and Goldstone-Malargue. The coverage (yellow area) of the two baselines at lunar distance is shown in Figure 1.

In this paper, we consider simultaneous ΔDOR or SBI of multiple baselines[2] that share one common ground station, and focus on efficient computation approach that integrates the measurements from multiple baselines. We show that when there are two or more baselines and when there is a known reference[3] in the vicinity of the spacecraft, precise 2-dimensional (2D) pointing vector between the common ground station and the spacecraft can be computed in real-time. Together with the standard deep space 2-way Doppler and ranging techniques, this method enables high-accuracy and real-time 3-dimension (3D) position determination. This is particularly useful for quick trajectory determination for spacecraft with electric propulsion, and for spacecraft of human missions that requires frequent trajectory corrections.

Note that in deep space approach of 3D positioning, range estimation and pointing vector determination are separate and independent processes. This avoids the problem that arises in direct application of Global Positioning System (GPS) style positioning methods to deep space where the baselines are a

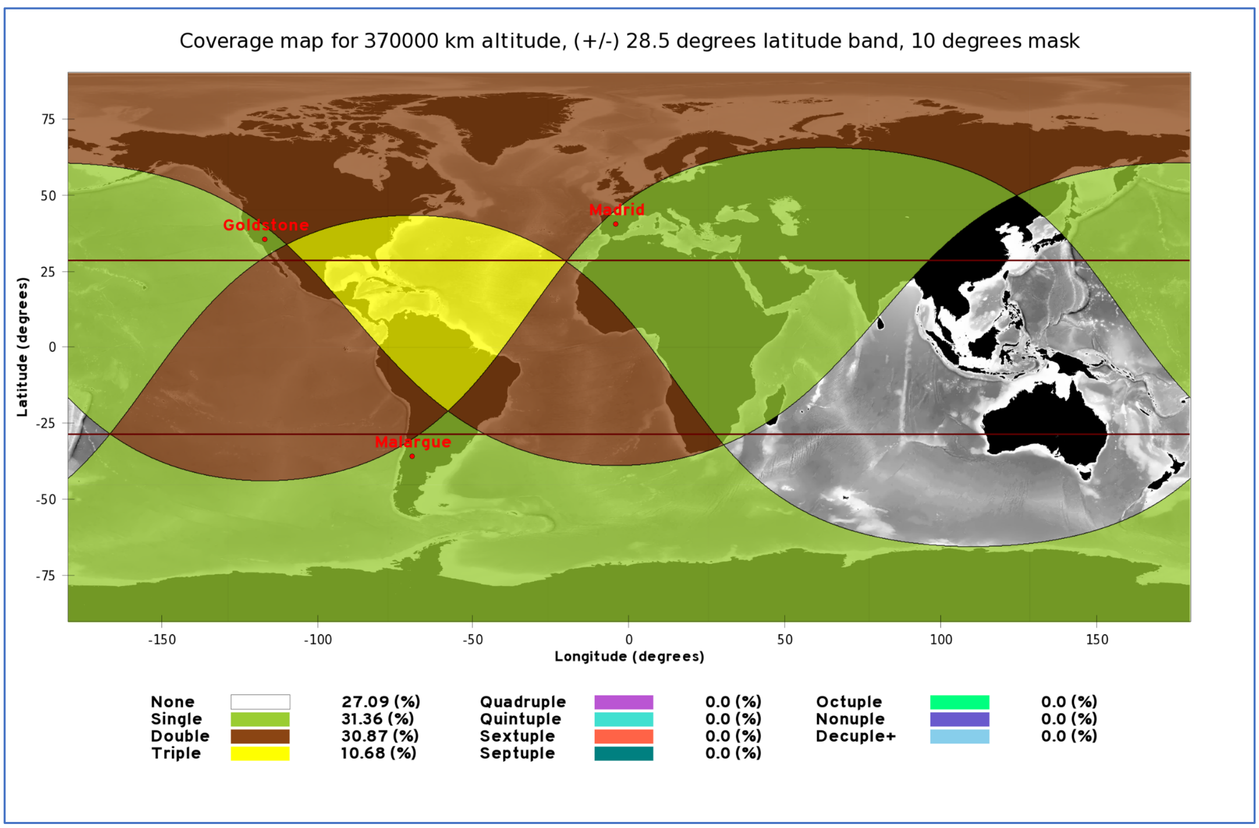


Figure 1. Simultaneous Coverage by Goldstone, Madrid, and Malargue at Lunar Distance

[2] Any two baselines are not collinear.

[3] A quasar or another spacecraft whose angular positions from various baselines are accurately known.

lot smaller compared to the range, thus resulting in a system that is highly sensitive to systematic and random errors. That is, a system with very high Position Dilution of Precision (PDOP). But unlike GPS-style infrastructures that broadcast navigation signals and support an infinite number of users, the methods described in this paper can support any number of users within the overlapping antenna beam patterns, but requires processing individual user's received signals separately.

As a corollary, we show that for the most common case of two baselines, the pairwise angular separations between the two baselines and the target spacecraft have to satisfy an inequality in order for the solution to exist. We also explore additional insight and outline efficient computation procedures for this case.

The double-differencing techniques used in ΔDOR and SBI can have near-Earth applications. We derive the theory and describe a high-level system concept that demonstrates the feasibility of detecting and locating dead and non-cooperatively spacecraft in the Geostationary Orbit (GEO). This can be done by making use of an existing GEO satellite as a reference, and/or by placing a dedicated "reference" spacecraft into an eccentric geosynchronous orbit over a region of interest (e.g. above North America). By adjusting the orbit, the "reference" spacecraft can loiter around the sky back-and-forth in the vicinity of the GEO over the region4. In this way, the reference spacecraft can be close to any "static" GEO targets along its path. Using the multi-static radar approach, the ground transmitting radar illuminates both the reference and target spacecraft, and the ground receiving radars measure the different time-delays of signal arrival. The different time-delays (double-differencing) can then be used to compute the precise relative position of the target spacecraft with respect to the reference spacecraft.

The rest of the paper is organized as follows: Section II discusses the ΔDOR and SBI techniques, and outlines a computation method that integrates simultaneous ΔDOR and SBI measurements from multiple baselines to estimate the precise pointing vector. Section III focuses on the special case of two baselines, derives the sufficient condition that guarantees the existence of a solution, and discuss the computation procedure. Section IV discusses the application of the powerful double-differencing techniques in ΔDOR and SBI for detecting and locating dead and non-cooperatively spacecraft in the GEO using a reference satellite and multi-static radar. Section V provides concluding remarks and discusses future work.

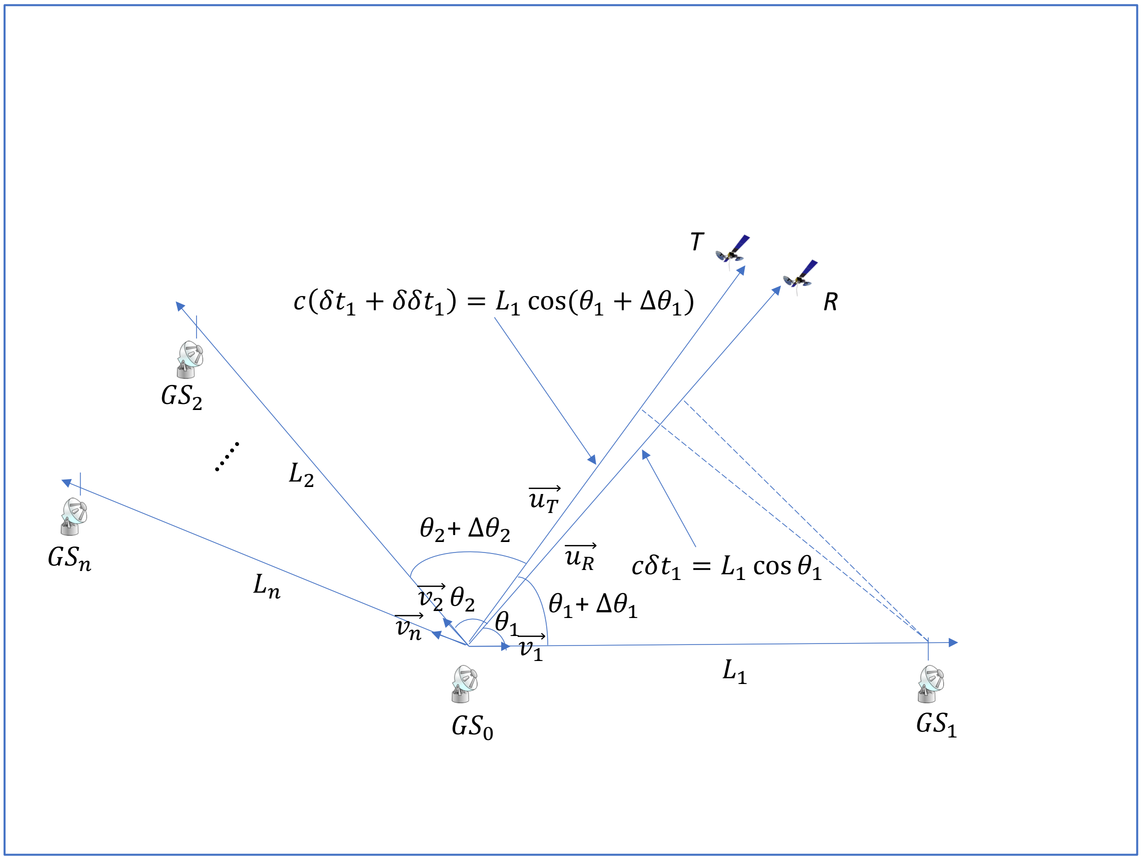


Figure 2. Navigation Tracking with Multiple Baselines

[4] This operation requires frequent orbit adjustment that consumes fuel, but on-orbit GEO satellite refueling service (estimated to be available by 2021) can extend the life of the satellite.

2. ΔDOR THEORY AND ITS EXTENSION TO SIMULTANEOUS BASELINES

Consider Figure 2 that includes a common ground station GS_0 that form n baselines with n ground stations GS_1, GS_2, … GS_n, with distances of separation between the common ground station and the n other stations to be L_1, L_2, … L_n respectively. The baselines' unit vectors are denoted by $\overrightarrow{v_1}, \overrightarrow{v_2}, \dots, \overrightarrow{v_n}$. The spacecraft R and T are assumed to be at a far distance (e.g. Mars distance), and their signals arrive at the ground stations are approximated as plane wave. The unit vectors along the line-of-sight directions between the common ground station GS_0 and spacecraft R and T are $\overrightarrow{u_R}$ and $\overrightarrow{u_T}$ respectively. The time delay of signal arrival from the reference spacecraft R to reach GS_0 compared to GS_i is denoted as δt_i, and the additional travel distance can be expressed as follows:

$$c\delta t_i \approx L_i \cos\theta_i \quad \text{for } i = 1, 2, \dots, n \tag{1}$$

where c is the speed-of-light, and θ_i is the angle between the baseline unit vectors $\overrightarrow{v_i}$ and $\overrightarrow{u_R}$ and is assumed to be accurately known. The time delay of signal arrival from the target spacecraft T to reach GS_0 compared to GS_i can also be denoted as $\delta t_i + \delta\delta t_i$, and the additional travel distance can be expressed as:

$$c(\delta t_i + \delta\delta t_i) \approx L_i \cos(\theta_i + \Delta\theta_i) \quad \text{for } i = 1, 2, \dots, n \tag{2}$$

where $\theta_i + \Delta\theta_i$ is the angle between the unit vector $\overrightarrow{v_i}$ and $\overrightarrow{u_T}$. Note that $\delta\delta t_i$ is the double difference of the delay measurements between the reference spacecraft and the target spacecraft. Taking difference between equations 1 and 2, and assuming small $\Delta\theta_i$,

$$c\delta\delta t_i \approx L_i \cos(\theta_i + \Delta\theta_i) - L_i \cos\theta_i$$

$$\cos(\theta_i + \Delta\theta_i) = \frac{c\delta\delta t_i + L_i \cos\theta_i}{L_i} + \epsilon_i \quad \text{for } i = 1, 2, \dots, n \tag{3}$$

where ϵ_i is a wave-front curvature correction factor[5]. The SBI method with a slightly different formulation, and its performance in tracking Magellan and Pioneer Venus Orbiters are discussed in details in [5].

To compute the 2-dimensional (2D) pointing vector or the directional cosine of $\overrightarrow{u_T}$, we have

$$\overrightarrow{u_T} \cdot \overrightarrow{v_i} = \frac{c\delta\delta t_i + L_i \cos\theta_i}{L_i} + \epsilon_i \quad \text{for} \quad i = 1, 2, \dots, n \tag{4}$$

Note that $\overrightarrow{u_T}$ is a unit vector, thus

$$\overrightarrow{u_T} \cdot \overrightarrow{u_T} = 1 \tag{5}$$

Next, we construct the cost function $f_i(\overrightarrow{u_T})$ as follows:

$$f_i(\overrightarrow{u_T}) = \overrightarrow{u_T} \cdot \overrightarrow{v_i} - \frac{c\delta\delta t_i + L_i \cos\theta_i}{L_i} - \epsilon_i \quad \text{for } i = 1, 2, \dots, n \tag{6}$$

The unit vector constraint is expressed as the cost function $f_U(\overrightarrow{u_T})$ as shown below:

$$f_U(\overrightarrow{u_T}) = \overrightarrow{u_T} \cdot \overrightarrow{u_T} - 1 \tag{7}$$

It is found that the unit vector constraint can be added to the cost functions $f_i(\overrightarrow{u_T})$, $= 1, 2, \dots, n$, to improve the convergence and accuracy of the calculations (equation 8). That is,

$$f_i'(\overrightarrow{u_T}) = f_i(\overrightarrow{u_T}) + \overrightarrow{u_T} \cdot \overrightarrow{u_T} - 1 \quad \text{for } i = 1, 2, \dots, n \tag{8}$$

The Jacobian of this cost function can be calculated (equation 9). For $\overrightarrow{u_T} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$,

$$J_{i1}(x, y, z) = \frac{\partial f_i}{\partial x} + 2x \tag{9a}$$

$$J_{i2}(x, y, z) = \frac{\partial f_i}{\partial y} + 2y \tag{9b}$$

$$J_{i3}(x, y, z) = \frac{\partial f_i}{\partial z} + 2\mathrm{z} \tag{9c}$$

The Jacobian matrix is a $(n + 1) \times 3$ matrix as shown below

$$J(\overrightarrow{u_T}) = \begin{bmatrix} \frac{\partial f_1}{\partial x} + 2x & \frac{\partial f_1}{\partial y} + 2y & \frac{\partial f_1}{\partial z} + 2\mathrm{z} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial z} + 2\mathrm{x} & \frac{\partial f_n}{\partial y} + 2y & \frac{\partial f_n}{\partial z} + 2\mathrm{z} \\ 2x & 2y & 2z \end{bmatrix} \tag{10}$$

We then evaluate $\overrightarrow{u_T}$ using the Newton's Method as shown below. As the target spacecraft T is in the vicinity of the reference spacecraft R, using an initial guess $\overrightarrow{u_{T,0}} = \overrightarrow{u_R}$, $\overrightarrow{u_{T,k}}$ converges to $\overrightarrow{u_T}$ (equations 11 – 12).

[5]A good estimation of ϵ_i is $-L_i\,(Sin\theta_i)^2/2r$, where r is an approximated range between GS_0 and T.

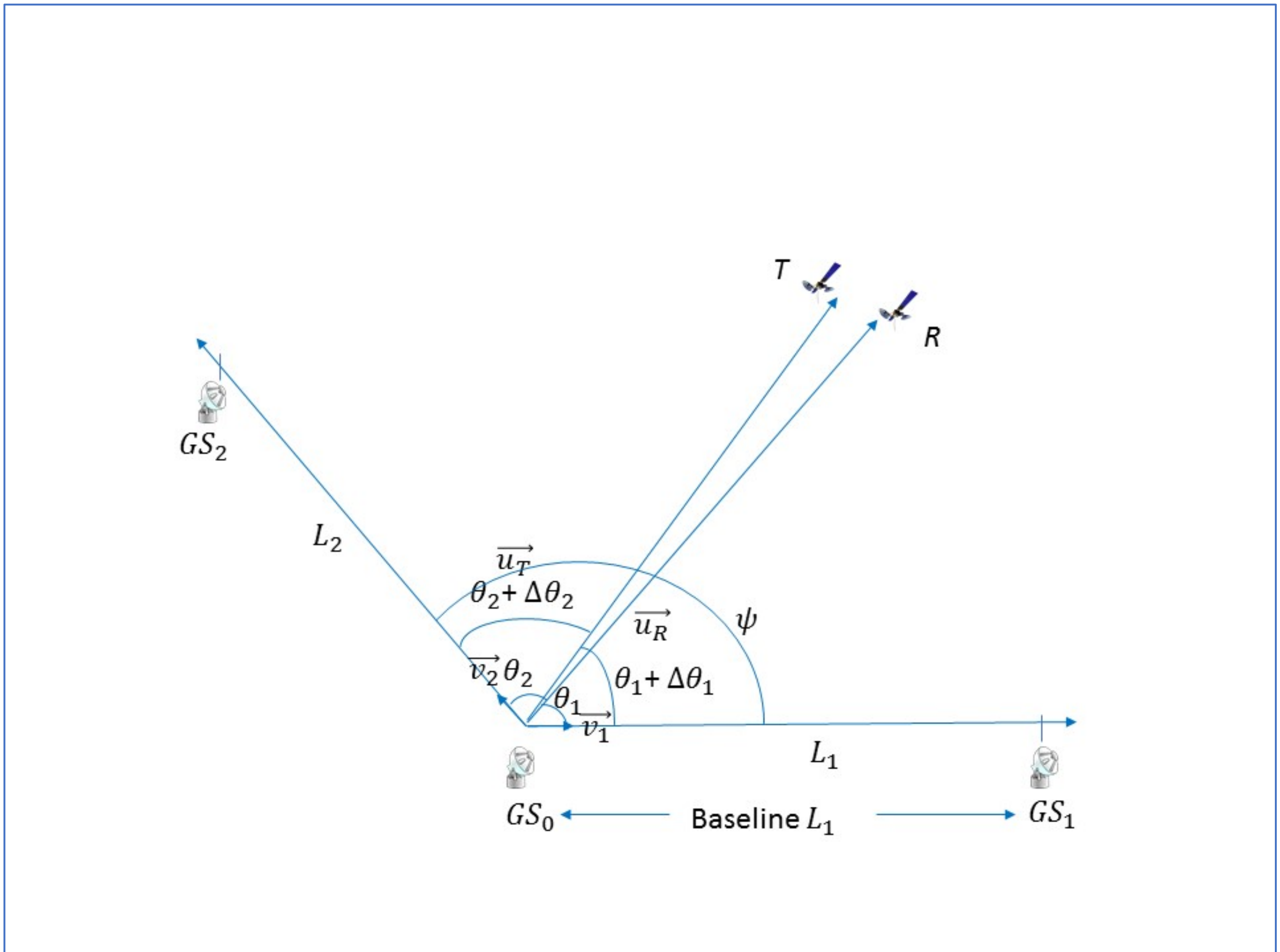


Figure 3. Geometry of Two Baselines

$$\overrightarrow{u_{T,0}} = \overrightarrow{u_R} \; F_0 = \begin{bmatrix} f_1'(\overrightarrow{u_{T,0}}) \\ \vdots \\ f_n'(\overrightarrow{u_{T,0}}) \\ f_U(\overrightarrow{u_{T,0}}) \end{bmatrix} \; J_0 = J(\overrightarrow{u_{T,0}}),$$

$$F_k = \begin{bmatrix} f_1'(\overrightarrow{u_{T,k}}) \\ \vdots \\ f_n'(\overrightarrow{u_{T,k}}) \\ f_U(\overrightarrow{u_{T,k}}) \end{bmatrix} \; J_k = J(\overrightarrow{u_{T,k}})$$

$$\Delta\overrightarrow{u_{T,k}} = \left({J_k}^T J_k\right)^{-1} {J_k}^T F_k \tag{11}$$

$$\overrightarrow{u_{T,k+1}} = \overrightarrow{u_{T,k}} - \Delta\overrightarrow{u_{T,k}} \tag{12}$$

3. Sufficient Condition and Computation Procedure for the Case of Two Baselines

For the case of two baselines, which is the minimal and probably the most common configuration to estimate the pointing vector in real-time, the geometry can be depicted as shown in Figure 3. Let ψ be the angle sustained between $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$. $\theta_1 + \Delta\theta_1$ and $\theta_2 + \Delta\theta_2$ be the angles sustained between $\overrightarrow{u_T}$ and $\overrightarrow{v_1}$ and between $\overrightarrow{u_T}$ and $\overrightarrow{v_2}$ respectively. Note that $\overrightarrow{u_T}$, $\overrightarrow{v_1}$, and $\overrightarrow{v_2}$ are not necessarily coplanar. To investigate the relationship between ψ, $\theta_1 + \Delta\theta_1$ and $\theta_2 + \Delta\theta_2$, we rewrite equation 4 as

$$\overrightarrow{u_T} \cdot \overrightarrow{v_1} = \cos(\theta_1 + \Delta\theta_1) \tag{13}$$

$$\overrightarrow{u_T} \cdot \overrightarrow{v_2} = \cos(\theta_2 + \Delta\theta_2) \tag{14}$$

Without loss of generality, we assign $\overrightarrow{v_1} = \begin{bmatrix}1\\0\\0\end{bmatrix}$. To simplify the derivation, we want $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ to span the x-y plane such that the normal vector to the plane (z-axis) is $\begin{bmatrix}0\\0\\1\end{bmatrix}$. From the fact that ψ being the angle sustained between $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$, $\overrightarrow{v_2} = \begin{bmatrix}\cos(\psi)\\\sin(\psi)\\0\end{bmatrix}$ and $\overrightarrow{v_1} \times \overrightarrow{v_2} = \begin{bmatrix}0\\0\\1\end{bmatrix}$. Let $\overrightarrow{u_T} = \begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}$. From equations 13 and 14, $u_x = \cos(\theta_1 + \Delta\theta_1)$ and $u_y = \frac{\cos(\theta_2+\Delta\theta_2)-\cos(\theta_1+\Delta\theta_1)\cos(\psi)}{\sin(\psi)}$. Substitute into equation 5, it can be shown that u_z can only have real value(s) if the following condition is met

$$1 + 2Cos(\psi)Cos(\theta_1 + \Delta\theta_1)Cos(\theta_2 + \Delta\theta_2) - Cos^2(\psi) - Cos^2(\theta_1 + \Delta\theta_1) - Cos^2(\theta_2 + \Delta\theta_2) \geq 0 \qquad (15)$$

Equation 15 provides the sufficient condition for a solution to exist for $\overrightarrow{u_T}$ in the case of two baselines. The two possible solutions of $\overrightarrow{u_T}$ are denoted as $\overrightarrow{u_{T+}}$ and $\overrightarrow{u_{T-}}$, and are given by

$$\overrightarrow{u_{T+}} = \begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix} = \begin{bmatrix} Cos(\theta_1 + \Delta\theta_1) \\ \frac{\cos(\theta_2+\Delta\theta_2)-\cos(\theta_1+\Delta\theta_1)\cos(\psi)}{\sin(\psi)} \\ +\sqrt{1-(\cos(\theta_1+\Delta\theta_1))^2 - \frac{(\cos(\theta_2+\Delta\theta_2)-\cos(\theta_1+\Delta\theta_1)\cos(\psi))^2}{(\sin(\psi))^2}} \end{bmatrix} \qquad (16a)$$

$$\overrightarrow{u_{T-}} = \begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix} = \begin{bmatrix} Cos(\theta_1 + \Delta\theta_1) \\ \frac{\cos(\theta_2+\Delta\theta_2)-\cos(\theta_1+\Delta\theta_1)\cos(\psi)}{\sin(\psi)} \\ -\sqrt{1-(\cos(\theta_1+\Delta\theta_1))^2 - \frac{(\cos(\theta_2+\Delta\theta_2)-\cos(\theta_1+\Delta\theta_1)\cos(\psi))^2}{(\sin(\psi))^2}} \end{bmatrix} \qquad (16b)$$

As the location of the reference spacecraft R is known (so is $\overrightarrow{u_R}$), the correct solution is one that is closer in angular distance with $\overrightarrow{u_R}$. That is, if $\overrightarrow{u_{T+}} \cdot \overrightarrow{u_R} > \overrightarrow{u_{T-}} \cdot \overrightarrow{u_R}$, then $\overrightarrow{u_T} = \overrightarrow{u_{T+}}$, and otherwise $\overrightarrow{u_T} = \overrightarrow{u_{T-}}$,.

This artificial assignments of $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ reveal additional geometric insight. Note that the two baselines of ground stations define the xy-plane on the Earth's surface and form a topographic frame of reference. The x- and y- components of $\overrightarrow{u_{T+}}$ and $\overrightarrow{u_{T-}}$ are identical, and the z-components represent the elevation angles, which have same magnitude and opposite signs. This is consistent with the mental intuition that the two solutions of pointing vector that satisfy equations (13), (14), and (5) are on the opposite side of the plane defined by $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$. So as along as the reference spacecraft R is at a reasonably high elevation angle with respect to the common ground station, the ambiguity of $\overrightarrow{u_{T+}}$ and $\overrightarrow{u_{T-}}$ can be easily resolved. This also justifies the choice of $\overrightarrow{u_R}$ to be the initial guess as input to the Newton's Method as discussed in Section II.

For the general case of arbitrary $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$, the computation procedure is outlined as follows:

1. Note that $\cos(\psi) = \overrightarrow{v_1} \cdot \overrightarrow{v_2}$. Check if the sufficient condition is satisfied (eqn 15).

2. Note that equations 13 and 14 can be viewed as two planes with normal vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ respectively. The two planes intersect to form a line, whose direction is given by the unit vector $\vec{l} = \frac{\overrightarrow{v_1} \times \overrightarrow{v_2}}{\|\overrightarrow{v_1} \times \overrightarrow{v_2}\|}$. Set $o_z = 0$, and solve for o_x and o_y using equations 13 and 14. The line passes through the xy-plane (z = 0) at $\vec{o} = \begin{bmatrix}o_x\\o_y\\0\end{bmatrix}$.

3. Given that $\overrightarrow{u_T}$ is a unit vector and if the solution(s) exit, they lie on the line $\vec{r} = \vec{o} + d\vec{l}$ and intersect with the unit sphere $\|\vec{r}\| = 1$. This requires satisfying the condition $(\vec{l}.\vec{o})^2 - \vec{o}.\ \vec{o} + 1 \geq 0$. It can be shown that this is equivalent to satisfying the sufficient condition in equation 15.

4. The scalar d is evaluated as $d = -\vec{l}.\vec{o} \pm \sqrt{(\vec{l}.\vec{o})^2 - \vec{o}.\ \vec{o} + 1}$.

5. Substitute the value(s) of d into the expression $\vec{r} = \vec{o} + d\vec{l}$ to obtain $\overrightarrow{u_{T+}}$ and $\overrightarrow{u_{T-}}$.

6. Determine the correct solution $\overrightarrow{u_T}$ by checking if $\overrightarrow{u_{T+}}$ or $\overrightarrow{u_{T-}}$ is closer in angular distance with $\overrightarrow{u_R}$.

4. LOCATING UNCOOPERATIVE SPACECRAFT AT GEO USING A REFERENCE SATELLITE AND MULTI-STATIC RADAR

In this section, we discuss the application of the powerful double-differencing techniques in ΔDOR and SBI for detecting and locating dead and non-cooperatively spacecraft in the GEO using a reference satellite and multi-static radar. Note that for tracking satellites at GEO distance by Earth's ground stations, the far-field assumption that signals arrive in plane wave does not apply.

A multi-static radar system consists of a number of spatially diverse transmitting radars and receiving radars. The spatial diversity allows different aspects of a target satellite to be viewed simultaneously. Each transmitting radar emits radar pulses towards the target and the reflected radar echoes are received by one or more receiving radars. A pair of transmitting and receiving radars form a bistatic radar system, and the ranges between the target and the transmitting and receiving radars can be estimated by the time of arrival of radar signals. Measurements from all bistatic radar pairs within the multi-static radar system are integrated together to estimate the target position [6][7]. The standard way of range estimation in a bistatic radar pair is based on the time difference δt_{rt} between receiver's reception of transmitted pulse (transmitter – receiver path length = L) and of the target echo (transmitter-to-target-to-receiver path length = $R_T + R_R$), which arrive at the receiver from different signal paths as shown in Figure 4 [6][6]. The range estimates using this approach can be contaminated by systematic noises like atmospheric delay, instrument noise, and time bias. 3D positioning using the straight-forward multi-static radar is therefore not too accurate, with Root Mean Square (RMS) error of the order of 100's of meters or kilometers.

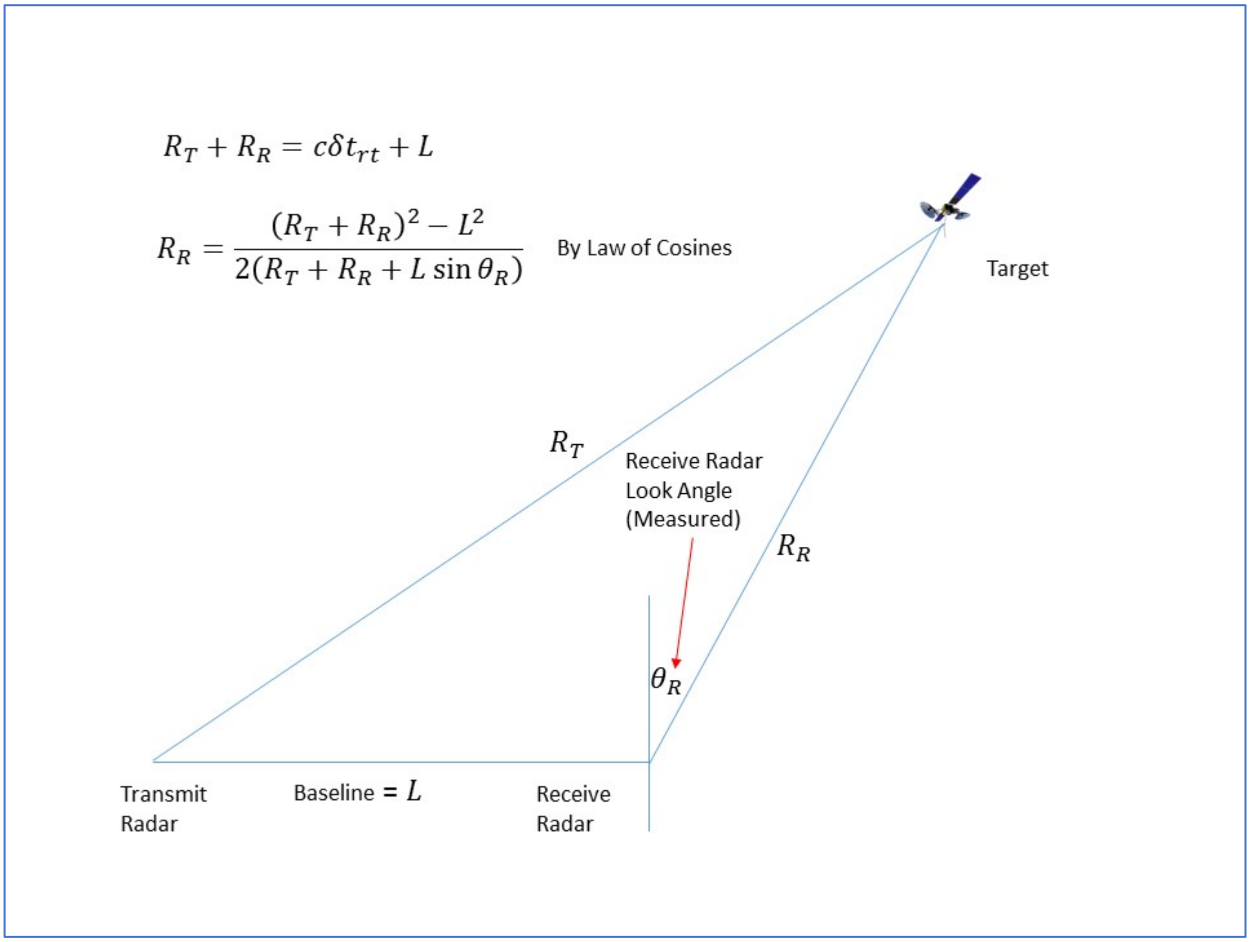


Figure 4. Range Estimation in Standard Bistatic Radar

When there is a reference satellite whose position is precisely known and is in the vicinity of the target satellite, one can use double-differencing of ranges to estimate the relative position between the target satellite and the reference satellite. This approach helps to cancel out or to greatly attenuate the systematic noises like atmospheric delay, instrument noise, and time bias, thus enables accurate 3D relative positioning of the target satellite with respective to the reference. In this section we describe a high-level system concept that illustrates the feasibility of detecting and locating dead and

[6] Note that [6] (Chapter 23) assumes the receiving radar can receive the radar pulse of the transmitting radar. If not, the receiving radar would need to know the transmission time of the radar pulse.

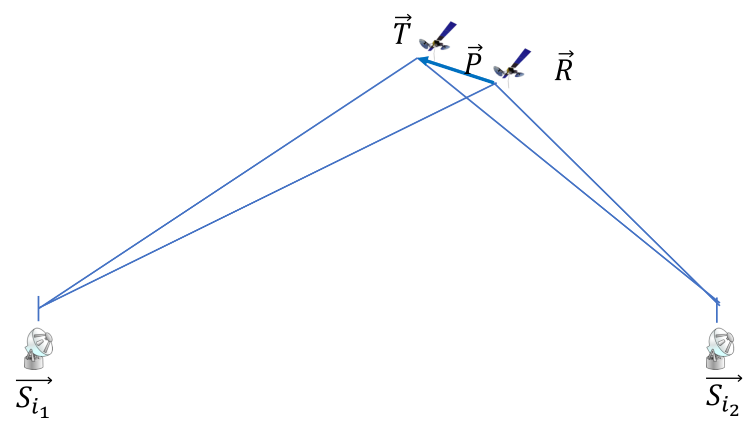
non-cooperatively spacecraft in the Geostationary Orbit (GEO). The concept includes making use of an existing GEO satellite with accurately known location, and/or placing a reference spacecraft into an eccentric geosynchronous orbit over a region of interest (e.g. above North America). By adjusting the orbit, the "reference" spacecraft can loiter around the sky back-and-forth in the vicinity of the GEO over the region[7]. In this way, the reference spacecraft can be close to any "static" GEO targets along its path. A ground transmitting radar in the multi-static radar system illuminates both the reference and target spacecraft, and the ground receiving radars measure the different time-delays of signal arrival. The differences of time-difference-of-arrival (TDOA), also known as double-differencing, can then be used to compute the precise relative position of the target spacecraft with respect to the reference spacecraft. The position of the reference spacecraft can be accurately estimated with meter-level accuracy using the weak Global Positioning Satellite (GPS) signals [8][9]. For this hypothetical analysis, we assume the ideal case that the radar echoes from the reference and target satellites can be accurately detected and identified.

The 3D positioning computation method is outlined as follows.

In this problem formulation that leverages on "double-differencing", n antennas form $n-1$ independent baselines. The multi-static radar system consists of one or more transmitting antennas[8] and n receiving antennas whose positions are accurately known, where $n \geq 4$. Consider taking a subset of m baselines for satellite position determination, where $3 \leq m \leq \binom{n}{2}$. The m baselines must include at least 3 independent baselines to achieve a 3D position fix. Consider the i^{th} baseline with two receiving antennas GS_{i_1} and GS_{i_2} shown in Figure 5.

Using a similar approach as in Section II on the key equation in Figure 5, we construct the cost function

$$f_i(\vec{P}) = c\delta\delta t_i - \left(\|\vec{R} - \overrightarrow{S_{i_1}}\| - \|\vec{R} - \overrightarrow{S_{i_2}}\|\right) - \left(\|(\vec{R} - \overrightarrow{S_{i_1}}) + \vec{P}\| - \|(\vec{R} - \overrightarrow{S_{i_2}}) + \vec{P}\|\right)$$
$$\text{for } i = 1, 2, \dots, m \qquad (17)$$



for $i = 1, 2, \dots, m \quad m \geq 3$

$\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, three unknown

Measurements of time-delay of arrival

$$c\delta\delta t_i = \left(\|\overline{S_{i_1}\vec{R}}\| - \|\overline{S_{i_2}\vec{R}}\|\right) - \left(\|\overline{S_{i_1}\vec{T}}\| - \|\overline{S_{i_2}\vec{T}}\|\right)$$
$$= \left(\|\vec{R} - \overrightarrow{S_{i_1}}\| - \|\vec{R} - \overrightarrow{S_{i_2}}\|\right) - \left(\|(\vec{R} - \overrightarrow{S_{i_1}}) + \vec{P}\| - \|(\vec{R} - \overrightarrow{S_{i_2}}) + \vec{P}\|\right)$$

[7] This operation requires frequent orbit adjustment that consumes fuel, but on-orbit GEO satellite refueling service (estimated to be available by 2021) can extend the life of the satellite.

[8] In this paper, we consider the general case of one or more transmitting antennas. In a future paper, we will show that if one transmitting antenna can illuminate both the reference and target spacecraft, a range equation can be set up using the measured TDOA between the reference and target spacecraft ("single-differencing") at each receiving radar. This can greatly improve the positioning accuracy performance.

where $\|.\|$ denotes the magnitude of a vector. For example, for the relative position vector $\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\|\vec{P}\| = \sqrt{x^2 + y^2 + z^2}$. Note that $\overrightarrow{S_{\iota_1}}$, $\overrightarrow{S_{\iota_2}}$, and $\vec{R}$ are known quantities. The Jacobian of this cost function can be calculated (equation 18) as follows:

$$J_{i1}(x, y, z) = \frac{\partial f_i(\vec{P})}{\partial x} \tag{18a}$$

$$J_{i2}(x, y, z) = \frac{\partial f_i(\vec{P})}{\partial y} \tag{18b}$$

$$J_{i3}(x, y, z) = \frac{\partial f_i(\vec{P})}{\partial z} \tag{18c}$$

The Jacobian matrix is a $m \times 3$ matrix as shown below

$$J\left(\vec{P}\right) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x} & \frac{\partial f_m}{\partial y} & \frac{\partial f_m}{\partial z} \end{bmatrix} \tag{19}$$

We then evaluate $\vec{P}$ using the Newton's Method as shown below. Using an initial guess $\overrightarrow{P_0}$,

$$\overrightarrow{P_0} = \begin{bmatrix} P_{o,x} \\ P_{o,y} \\ P_{o,z} \end{bmatrix} \quad F_0 = \begin{bmatrix} f_1(\overrightarrow{P_0}) \\ f_2(\overrightarrow{P_0}) \\ \vdots \\ f_m(\overrightarrow{P_0}) \end{bmatrix} \quad J_0 = J(\overrightarrow{P_0}), \qquad F_k = \begin{bmatrix} f_1(\overrightarrow{P_k}) \\ f_2(\overrightarrow{P_k}) \\ \vdots \\ f_m(\overrightarrow{P_k}) \end{bmatrix} \quad J_k = J(\overrightarrow{P_k})$$

$$\Delta\overrightarrow{P_k} = \left(J_k{}^T J_k\right)^{-1} J_k{}^T F_k \tag{20}$$

$$\overrightarrow{P_{k+1}} = \overrightarrow{P_k} - \Delta\overrightarrow{P_k} \tag{21}$$

5. Conclusion and Future Work

In this paper we describe how the operational ΔDOR or SBI deep space tracking techniques use double-differencing in a single baseline and a reference source to eliminate the systematic biases in the received deep space signals, and extend these tracking techniques to simultaneous and multiple baselines that share one common ground station. We introduce an iterative computation method that integrates measurements from multiple baselines to estimate the 2D pointing vector between the common ground station and the spacecraft in real-time. Together with the standard deep space 2-way Doppler and ranging techniques, this method enables high-accuracy and real-time 3D position determination. For the case of two baselines, we derive the necessary condition for the geometry between the spacecraft and the ground antennas in order for a solution to exist.

We also describe a variant of the above deep space tracking techniques for near-Earth applications. We illustrate this approach using the hypothetical scenario of detecting and locating dead and non-cooperatively spacecraft in the GEO. The system concept involves placing a "reference" spacecraft into an eccentric geosynchronous orbit over a region of interest (e.g. above North America), and using a multi-static radar system that transmits the radar pulses to the reference and target spacecraft. The receiving antennas receive the radar echoes and measure the differences of time delays of signal arrival (double-differencing). The "pristine" double-differencing measurements from different baselines can then be used to estimate the precise position of the target spacecraft relative to the reference.

The plan for path-forward is as follows:

1. In an upcoming paper (Part 2), we plan to perform in-depth simulations for the deep space and near-Earth scenarios as described in Section II and IV. We will evaluate the 3D positioning performance under reasonable error conditions that include ephemeris error of the reference satellite and the ground antennas, random pseudo-range measurement errors, media delays (e.g. solar plasma, atmosphere, etc.) and clock biases between the reference spacecraft, the target spacecraft, and the ground antennas.

2. We plan to investigate the challenges of radar detection and target identifications at GEO and lunar distances, and to apply the techniques described in Sections IV to detect debris and/or uncooperative spacecraft that orbit around the Moon using simultaneous antenna baselines on Earth. The reference can be a fixed prominent feature (e.g. a crater) on the near-side of the lunar surface that can provide distinct radar echo signature, and/or a dedicated beacon.

3. Note that in Section 4, we consider the general case of one or more transmitting radars. We will show that if one transmitting radar can illuminate both the reference and the target spacecraft, a range equation can be established using the measured TDOA between the reference and target ("single-differencing") at each receiving radar. This "single-differencing" approach would provide much better accuracy performance.

4. The techniques described so far are exact analytical methods for real-time positioning. We plan to consider the use of Kalman filters to improve the robustness and performance.

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